2.6 Trig and limits

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$$\lim_{x\to 0}\frac{1-\cos x}{x}=\lim_{x\to 0}\frac{1-\cos x}{x}\cdot\frac{1+\cos x}{1+\cos x}$$

$$=\lim_{x\to 0}\frac{1-\cos^2 x}{x(1+\cos x)}=\lim_{x\to 0}\frac{\sin^2 x}{x(1+\cos x)}$$

$$=\lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{\sin x}{1+\cos x}$$

$\frac{\sin x}{x}\to 1$

$\frac{\sin x}{1+\cos x}\to 0$, $1+1=2$

$$=\frac{0}{2}=0$$

Big Trig Equality

$\sin^2 x+\cos^2 x=1$

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27) $\lim_{x\to 0} \frac{x^2}{1-\cos x} \cdot \frac{1+\cos x}{1+\cos x} = \lim_{x\to 0} \frac{x^2(1+\cos x)}{1-\cos^2 x}$

$= \lim_{x\to 0} \frac{x^2(1+\cos x)}{\sin^2 x} = \lim_{x\to 0} \frac{x}{\sin x} \cdot \frac{x}{\sin x} \cdot \frac{1+\cos x}{1}$

$\frac{x}{\sin x} \to 1$, $\frac{x}{\sin x} \to 1$, $\frac{1+\cos x}{1} \to \frac{1+1}{1} =$

$= 2$

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20) $\lim_{x\to 0} \frac{\sin^2 x}{3x^2} = \lim_{x\to 0} \frac{1}{3}\cdot\frac{\sin x}{x}\cdot\frac{\sin x}{x} = \frac{1}{3}$

($\frac{1}{3}$ → $\frac{1}{3}$; $\frac{\sin x}{x}$ → 1; $\frac{\sin x}{x}$ → 1)

19) $\lim_{x\to 0^-} \frac{\sin x}{|x|}$

$= \lim_{x\to 0^-} \frac{\sin x}{-x} = -1$

($\frac{\sin x}{-x}$ → 1)

$|x| = \begin{cases} x\,; & x \geq 0 \\ -x\,; & x < 0 \end{cases}$

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24) $\lim_{x \to 0} \frac{\sin^2 x}{x} = \lim_{x \to 0} \frac{\sin x}{x} \cdot \sin x = 0$

($\frac{\sin x}{x} \to 1$, $\sin x \to 0$)

25) $\lim_{x \to 0} \frac{x}{\tan x} = \lim_{x \to 0} \frac{x}{\frac{\sin x}{\cos x}} = \lim_{x \to 0} \frac{x \cos x}{\sin x}$

$= \lim_{x \to 0} \frac{x}{\sin x} \cdot \frac{\cos x}{1} = 1$

($\frac{x}{\sin x} \to 1$, $\frac{\cos x}{1} \to 1$)

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33) $\lim_{x \to 0^+} \cos\left(\frac{1}{x}\right) = \cos\left(\lim_{x \to 0^+} \frac{1}{x}\right)$

$= \cos(\text{infinity-ish}) = \text{DNE}$ (because of oscillation)

$\lim \sqrt{\quad} = \sqrt{\lim}$